Kuwait University   Mth. 101   Feb 19, 1995

Math. Department   Final Exam (incomplete)   Duration: 2 Hrs.

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Answer the following questions. Q_1 to Q_6 6 Marks each and
Q_7 14 Marks.   Calculators are not allowed.

1) Evaluate each of the following limit if it exists:-

    a) $\lim\limits_{x \to 1} \dfrac{\sqrt{3 + x^2} - 2}{x - 1}$ b) $\lim\limits_{x \to 0} \dfrac{\tan 5x}{\sin 3x}$

2) a) For a function $f(x) = Ax^2 + Bx + C$ with $A \neq 0$ on the interval $[a,b]$.
     Show that the number c in (a,b) determined by the mean value
     theorem is the midpoint of the interval.

   b) Find the equation of the tangent line to the curve
       $1 + 16 \ x^2 y \ = \ \tan(x - 2y)$ at the point $(\frac{\pi}{4}, 0)$.

3) a) Evaluate the following integral $\displaystyle\int_{-\pi}^{\pi} x^2 \sin 2x \ dx$.

   b) Find the second derivative with respect to x of the function

     $y = \displaystyle\int_{0}^{x^2} \dfrac{dt}{(t^3 + 1)}$ at $x = 1$.

4) Evaluate   a) $\displaystyle\int \dfrac{dx}{\sqrt[5]{(8 - 3x)^6}}$ b) $\displaystyle\int_{0}^{3} \left|(x^2 - x - 2)\right| \ dx$

5) a) Given that $2x^2 - 8 \ = \ \displaystyle\int_{a}^{x} f(t) dt$. Find a formula for $f(x)$ and evaluate a

   b) If the average value of $f(x) \ = \ x^3 + bx - 2$ on $[0,2]$ is 4 , find b.

6) Find the dimensions of the right circular cylinder of maximum volume that
    can be inscribed in a right circular cone of redius 6 cm. and height 4 cm.

7) Let $f(x) = x + \dfrac{1}{x}$

   a) Find the intervals on which f is increasing or decreasing, and the
      local axtrema, if any.

   b) Find the intervals on which f is concave upward or downword,
      and the points of inflection, if any.

   c) Find the vertical and horizontal asymptotes for the graph f, if any

   d) Sketch the graph of the function.